This document will not simulate the actual final, but it is meant moreso as a comprehensive review!

1. Your company is engineering a new type of airplane wing for manufacturing that is being constructed according to the curve $y = e^x \sin(x)$ on the interval $[0, \pi]$. As a member of the engineering team, your job is to determine the amount of material (in square meters) that is needed to cover the surface of the wing.

2. A physicist comes to you and asks for your help as a Calc 2 student! He found that the area that particles cover over a certain time period behave according to the equation

$$A(t) = 2\int \frac{\sin(t)\sec^5(t) - \sin(t)\sec^3(t)}{\sin(2t)\cot(t)\csc^2(t) - \sin(2t)\cot(t)}\,dt$$

He can plug this into *Mathematica* and get an answer, but unfortunately he needs to know a few intermediate steps too for data purposes. Determine the time dependent area equation that the physicist needs, showing all your steps (and not leaving it to a computer)!

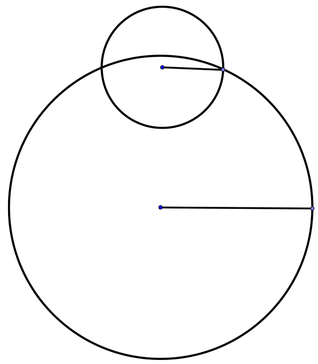
3. Determine the area under the curve, knowing that $x^4 - 6x^3 + 10x^2 - 6x + 9 = (x^2+1)g(x)$

$$y = \frac{7x^3 - 20x^2 + 5x + 16}{x^4 - 6x^3 + 10x^2 - 6x + 9}$$

(a) on the interval $[4, 10]$

(b) on the interval $[4, \infty]$

4. Consider the following picture of two circles, the smaller one of radius r and the larger of radius R. Determine the area that is inside the smaller circle, but outside of the larger circle. (This crescent shaped region is called a *lune*).



5. Find R_4, L_4, M_4, and T_4 for $\int_4^6 \ln\left(x^3+2\right)dx$. Label each as an over or underestimate.

6. State why the integral cannot be evaluated in its current form. Find a method to evaluate it and conclude either convergence (and stating its value) or divergence.

$$\int_4^{12} \frac{1}{\sqrt{x^2-16x+48}}\,dx$$

7. A local garage is creating a new gasket for an engine. After doing some modeling of the gasket, they find that the function $f(x) = \frac{1}{6}x^3 + \frac{1}{2x}$ from $x = \frac{1}{2}$ cm to $x = 3$ cm best models the edge of the gasket.

(a) How much material is required to create the edge of the gasket?

(b) As they begin production, the rest of the gasket is made by revolving it around the y-axis. How much material is required to produce one gasket?

8. The Panama Canal, approximately 48 miles in length, has a series of locks that ships must pass through in order to get through the canal. The water in the lock is roughly in the shape of an isosceles trapezoid with its width at the top being 110 feet wide (not including 10 feet on each side for the thickness of each wall) and its width at the bottom of the lock being 30 feet wide (as each wall is 50 feet thick). Knowing that the height of the lock is 66 feet, find the hydrostatic force on the lock when it is 2/3 full of water.

9. Consider the functions defined by

$$f(x) = \begin{cases} -3x, & \text{if } -2 \leq x < 0 \\ x^4, & \text{if } 0 \leq x < 2 \end{cases} \quad \text{and } g(x) = x^3 - x^2 + 1 \text{ and } h(x) = x^3 - 6x + 6$$

Find (a) the center of mass of $f(x)$, bounded by $y = 0$ (b) the center of mass of the region bounded by $g(x)$ and $h(x)$.

10. Consider the differential equation

$$\frac{dy}{dt} = -y^3 + 11y^2 - 30y$$

(a) What are the constant solutions?

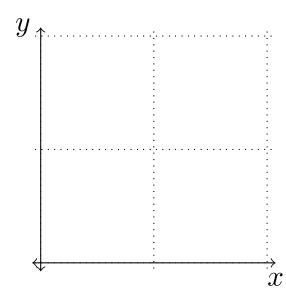
(b) When is y increasing? decreasing?

(c) Determine all solutions to the differential equation when $y(1) = 2$. NOTE: you will only be able to get the differential equation separated so that y is on one side and t is on the other. You will not be able to get $y = ...$

11. Consider the differential equation

$$y' = x^2y^2 - 2x - 3y + 1$$

(a) Sketch the direction field for all integer pairs (x, y) shown on the graph below.



(b) Using a step size of 0.05, determine three approximate solutions of the differential equation starting at the point $(x, y) = (0, 1)$

12. At the Natatorium in the U of M Rec Center, the pool there contains approximately 650,000 gallons of a water-chlorine mixture. Before admitting people to swim in the pool, the maintenance personnel ensure that there are 10,000 pounds of chlorine in the pool based on measurements and calculations. After allowing people to swim in the pool they add water-chlorine mixture, containing 1 pound of chlorine per 12 gallons, to the pool constantly at a rate of 20 gallons per minute. They also drain water off to prevent overflow at a rate of 15 gallons per minute. How much chlorine is in the pool one hour after admitting people into the pool?

13. Determine the Cartesian equation associated to the Parametric curves. Include a sketch and indicate the direction in which the curve is being traversed (i.e. as the parameter increases, which way does the curve go?).

$$x = \sqrt{t-2},\ y = e^t + 7,\ t \geq 2$$

14. Determine the parametric equations for a particle that follows the circle

$$(x+3)^2 + (y-5)^2 = 4$$

and starts at $(-5, 5)$, traverses clockwise, and ends at $(-3, 3)$.

15. In a forest, the fox and rabbit population can be modeled by the equations

$$\frac{dx}{dt} = 2x - 0.01xy$$

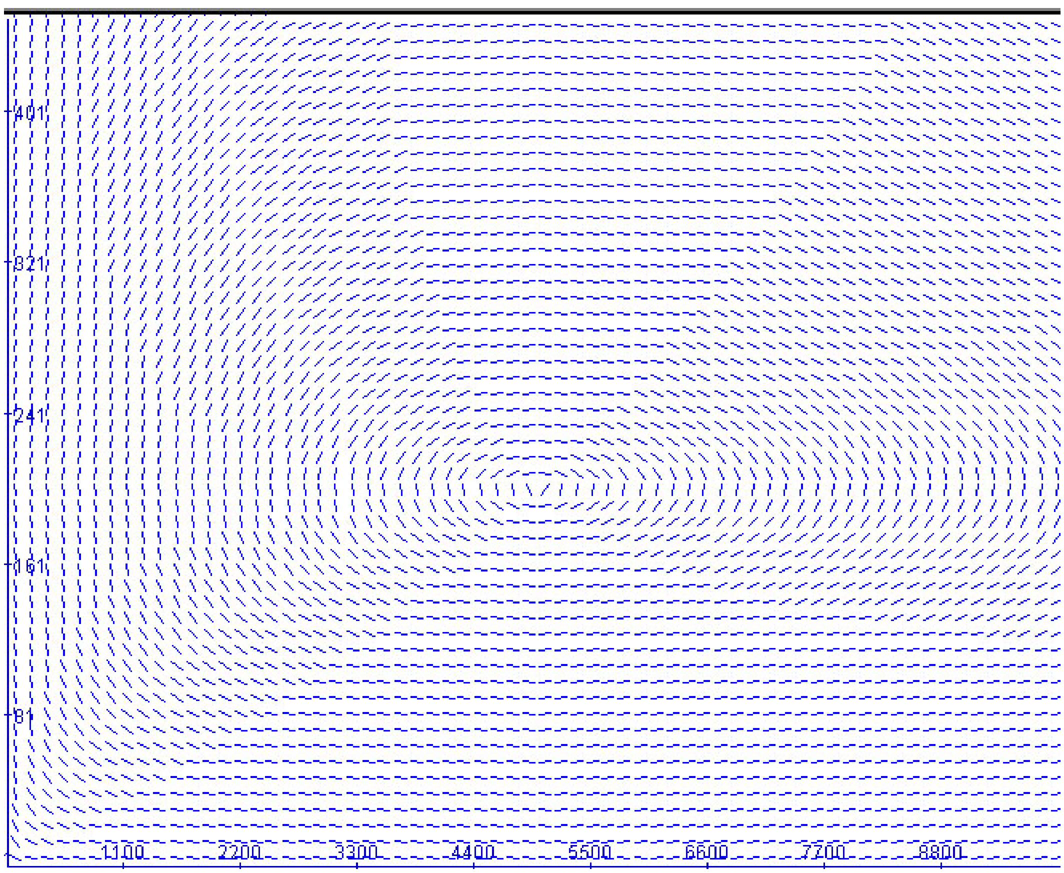
$$\frac{dy}{dt} = -0.8y + 0.0002xy$$

(a) Which variable represents foxes? rabbits? why?

(b) What are the equilibrium solutions?

(c) Determine an expression for $\frac{dy}{dx}$

(d) Sketch a phase portrait when there are 100 foxes and 5000 rabbits on the direction field below.



(e) Describe what happens to the fox and rabbit populations over time.

16. Consider the parameterization

$$x = -3t - t^3 \text{ and } y = 3t^2$$

(a) Determine all points where the tangent line has slope 1.

(b) Determine the length of the curve from $t = 3$ to $t = 8$.

17. A cissoid of Diocles is given by the equation

$$r = \sin(\theta)\tan(\theta)$$

(a) Determine the Cartesian equation for this curve.

(b) Determine at what values of x there are vertical tangents.

(c) Compute the Polar area between the cissiod and the cardioid $r = 4(1 - \cos(\theta))$. NOTE: the numbers here aren't pretty, but they are manageable...

18. Determine the area inside the curve $r = \sin(3\theta)$ and outside the curve $r = \cos(3\theta)$

19. Consider the sequences

$$a_n = \frac{\cos^2(n)}{2^n}, \; b_n = \frac{(-1)^{n-1}n^3}{n^3 + 2n^2 + 1}, \; c_n = \frac{(\ln(n))^2}{n}$$

(a) Do the above sequences have limits? If so, find them; if not, show divergence.

(b) Are the sequences bounded? Show your work using the textbook's definition of bounded.

(c) Determine which sequences are monotonically increasing, monotonically decreasing, or neither.

20. Prove convergence or divergence of the series.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln(n)}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{n^3+2n^2+5}$

(c) $\displaystyle\sum_{k=1}^{\infty} \frac{5^k}{3^k+4^k}$

(d) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k\sqrt{k}}{k+5}$

(e) $\displaystyle\sum_{k=1}^{\infty} \frac{(-2)^k k!}{(k+2)!}$

21. Determine the radius of convergence (ROC) and the interval of convergence (IOC) for the following power series:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^2 5^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n (x-3)^n}{\sqrt{n+3}}$

22. Find a power series representation of the functions

(a) $f(x) = \ln\left(\dfrac{1+x}{1-x}\right)$

(b) $f(x) = \dfrac{4}{x^2 - 12x + 32}$

23. Using power series, approximate the definite integral to six decimal places

$$\int_0^{0.5} 4x \sin\left(2x^3\right) dx$$

24. Determine the Taylor series for $f(x) = 1/\sqrt{x}$ centered around $x = 4$.

25. Write out the first three nonzero terms in the Maclaurin series for the function $f(x) = e^x \arctan(x)$

26. Find the sum of the series

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^2 + 9n + 18}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n e^{4n}}{729^n (2n)!}$

27. Let

$$\vec{r} = 3\hat{\imath} + 5\hat{\jmath} - 6\hat{k},\ \vec{s} = -4\hat{\imath} + \hat{\jmath} + 2\hat{k},\ \vec{t} = 6\hat{\imath} + 14\hat{\jmath} + 5\hat{k}$$

(a) Compute the angle formed by $\vec{r}$ and $\vec{s}$ as well as the angle formed by $\vec{s}$ and $\vec{t}$. If the angle is $\pi/2$, what does that mean about the vectors?

(b) Find a vector that is perpendicular to both $\vec{r}$ and $\vec{t}$.

YAY MATH!